Problem Set #5
EECS 381/409, Prof. Michael S. Branicky

**Reading Assignment:** Class notes, various on HyTech website

**Problem 5.1**
Consider the initialized multirate automaton depicted in Figure 1 (on the next page). Reduce it to a timed automaton using the algorithm outlined in class.

**Problem 5.2**
Consider the initialized multi-rectangular automaton depicted in Figure 2 (on the next page). Reduce it to an initialized multirate automaton using the algorithm outlined in class.

**Problem 5.3**
Consider the following one-dimensional game of cat and mouse. A mouse and a cat are initially in the same position, say zero, with the mouse running at a velocity $v_m$ towards its hole, located at $h$. At some time, $\delta$, later, the cat begins running toward the mouse at speed $v_c$. We will say that there are two possible outcomes: if the mouse reaches the hole before the cat, then the mouse wins. Otherwise, the cat wins. Draw an automaton modeling this situation. Make sure to include states denoting who has won.

Also, derive conditions on $v_c$, in terms of the other given variables, such that the cat wins. You may assume $v_c > v_m > 0$.

**Problem 5.4**
Consider the linear hybrid automaton of Figure 3 (on the next page). Draw the sets of reachable $(x, y)$ and $(y, z)$ pairs, respectively, for $y \leq 12$. If $y$ is time measured in minutes, how little (long) can the furnace have been on after $K$ hours?

**Problems 5.5–6**
The computer tool HyTech provides for automatic verification of embedded systems modeled as hybrid automata. An extensive set of resources is available at

http://embedded.eecs.berkeley.edu/research/hytech/ (link on webpage)

For example, there are papers, a user guide, executables for various platforms, and source code. In class, we handed out a set of hybrid automata for a train gate and for a mutual exclusion protocol. These files are available for download on the course webpage.

In these two problems, you will modify the HyTech code for these examples (which you just edit) to answer various questions regarding the these two systems.

*Note: Instructions for obtaining/accessing HyTech appear on the course website.*
**Problem 5.5.** We wish to ensure mutual exclusion once we add a third process, \( P_3 \), whose clock is inexact with real-time up within the interval \([4/5, 6/5]\).

Turn in your code for process \( P_3 \) (which you can cut-and-paste from the demo’s input window). What modifications did you have to make to the declarations? to the other process models? to the initial regions? to the final regions? Report the conditions on \( a \) and \( b \) for the system to satisfy mutual exclusion. (Do not turn in the full HyTech output!)

**Problem 5.6.** For this problem, we want to explore the “maximal safety margin” (in terms of distance) we can provide for the train gate controller. More specifically, the system in class and on the demo must have the gate down within 10 meters. We wish it to be down within \( x_{\text{max}} \) meters where \( x_{\text{max}} \) is as big as possible. Modify the demo’s train gate example to find \( x_{\text{max}} \). Briefly comment on your approach, the changes you made to the code, and how you verified (or would verify) the answer using HyTech (including only enough of its output to prove your point).