EECS 381/409: Problem Set #4

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Reading Assignment: Class notes. Alur and Dill: pp. 1–24, 42-44. Maler and Yovine.

Notes: Problems 4.1, 4.2, 4.5, and 4.3 can be done after Lecture #15: Omega Automata; Problems 4.3, 4.7, 4.1, and 4.2 can be done after Lecture #16: Timed Automata.

Problem 4.1

Consider the following transition table, where $\Sigma$ is the input alphabet, $S$ is the set of states, $S_0$ the set of initial states, and $E$ the set of edges:

$$\Sigma = \{a, b\}, \quad S = \{s_0, s_1\}, \quad S_0 = \{s_0\},$$
$$E = \{\langle s_0, a, s_1 \rangle, \langle s_0, b, s_0 \rangle, \langle s_1, a, s_1 \rangle, \langle s_1, b, s_0 \rangle\}.$$

(a) Draw its corresponding graph (i.e., automaton).
(b) Compute the language accepted by each Büchi automaton that can be associated with it.
(c) Compute the language accepted by each Muller automaton that can be associated with it.
(d) In general, if $|S| = n$, how many Büchi and Muller automata can be associated, respectively?

Problem 4.2

Consider the following transition table:

$$\Sigma = \{a, b\}, \quad S = \{s_0, s_1, s_2\}, \quad S_0 = \{s_0\},$$
$$E = \{\langle s_0, s_1, a \rangle, \langle s_0, s_0, b \rangle, \langle s_1, s_1, a \rangle, \langle s_1, s_2, b \rangle, \langle s_2, s_1, a \rangle, \langle s_2, s_0, b \rangle\}.$$

(a) Draw its corresponding graph (i.e., automaton).
(b) Compute the language accepted by the three Büchi automaton that can be associated with it where $F$ is a singleton, i.e., $F = \{s_0\}, \{s_1\}$, and $\{s_2\}$, respectively.
(c) Compute the language accepted by the seven Muller automaton that can be associated with it whose acceptance families are singletons: $F = \{F'\}$, where $F'$ is a non-empty subset of $S$.
(d) In general, if $|S| = n$, there are $n$ such Büchi and $2^n - 1$ such Muller automata. How can the languages of the remaining Büchi and Muller automata be computed from the languages of just these?
Problem 4.3
Consider the timed transition table depicted in Figure 3 of Alur and Dill. Modify it so that the time between any two successive symbols is less than 2. Also, formally state this timed language language in set notation (e.g., as in the last line on page 7). What is its untimed version?

Problem 4.4
Write out the formal run of the example timed word of Example 3.16 of Alur and Dill.

Problem 4.5
Recall Problem 2.13 of Cassandras and LaFortune (and Problem 1.7 for this class). Formally define and draw a Muller automaton which characterizes proper behavior when the timeout for job 1 is 2 and the timeout for job 2 is 3.

Problem 4.6
Draw the two region automata associated with both automata of Problem 4.3 (Figure 3 and yours).

Problem 4.7
Draw the four-state timed automata modeling a transistor with both rise-time and fall-time delays that is described in the Remark on page 3, column 2 of Maler and Yovine.

Problem 4.G1
Is the following timed word a member of $L_{\text{converge}}$? [See Alur and Dill, p. 13.]

$$(\sigma_i, \tau_i) = \begin{cases} (a, i), & i \text{ odd}, \\ (b, i + e^{-i/2}), & i \text{ even}. \end{cases}$$

Problem 4.G2
Consider the timed automaton of Figure 3 of Alur and Dill. Consider another, except with the clock constraint ($x > 1$)? replacing the one on the bottom edge. Formally following the proof of Theorem 3.15, construct an automaton accepting the intersection of their two languages. Then simplify it.

Problem 4.G3
Formally construct (following the proof of Theorem 3.20 of Alur and Dill) the Muller automaton (MA) and Büchi automaton (BA) equivalent to the BA and MA of Figures 1 and 2, respectively.